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Resolution of the Richards equation for the water transport in a growing substrate.

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Abstract

The soil less crop substrates should be a starting point for management models that can be used to obtain growth systems, with minimal input of water, nutrients, and pesticides, with a high production of good quality and with a minimal nuisance to the environment. These systems are better defined and controllable. For describing the water movement in a growing substrate, the h based form of Richards equation in two dimensions is obtained by combination of continuity equation and Darcy's law, this equation is implicitly solved using the control volume finite element method. The hydraulic conductivity, the water retention characteristic, and the differential moisture capacity have been described by the mathematical models known. A numerical model is developed for this aim and validated, by comparing the results of water transport with the numerical results obtained by CFD which are validated by experimental results. A growing substrate with specific dimensions, two drippers on top of the planting and the drain at lower right corner is tested by this model.

Key words: Richards equation, numerical model, water transport, growing substrate.

1. Introduction

For describing water movement we can proceed to: an experimental solution, an analytical solution or a numerical solution. Numerical solutions are more flexible and consider more complicated situations. For this purpose several codes were elaborated: HYDRUS [1] used finite element method, FUSSIM [2] used the control volume finite element method and [3] transformed the Navier- Stockes equations used by the

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CFD software [4] to the standard transport equations using the hydraulic and physical proprieties of the rockwool slab determined by [5]. In the same context [6] analyzed the solution motion in a rockwool slab by the model of sources and sinks which is simple and the results obtained are validated the experimental ones.

In this study we formulate a two dimensional numerical model for describing water movement, it is used to solve implicitly the Richards equation [7] for unsaturated porous medium with the control volume finite element method, giving water transport, followed by the general expressions of boundary and initial conditions. We compared and validated the results of water transport developed by this model with the numerical results obtained by CFD [3] which are validated by experimental results.

2. Materiel and methods

2.1. Rockwool

Rockwool is a chemically inert substrate, obtained from diabase, a volcanic rock, and has a porosity of about 96% [8]. The raw material is melted in large furnaces at 1600° until it becomes fluid lava. The fluid lava stream is dropped onto rapidly rotating disks, spinning the rockwool fibres. Calcium carbonate and other additives are added to the melted rock to adjust the physical properties of the final product.

Rockwool is mainly applied as a thermal insulation material, although it is also used as an acoustic insulation material, and as reinforcement for synthetic materials. At the end of 1960s, the use of rockwool as a growing medium was invented in Denmark [9] and produced by the Grodan Company. Rockwool slab is an artificial substrate, which offers a high degree of water permeability and high water content at low suction. A wide variety of substrates can be manufactured, differing mainly in the density and orientation. The Floriculture slab density used in this study, reached 0.0675kg/dm^3 and a porosity of 96.9%. The dimensions are: 7.5cm depth, 100cm length. Two drippers are located on top of the planting and the drain is located at lower right corner. Geometry, position of the drippers and drainage slot are recapitulated in see Fig 1.

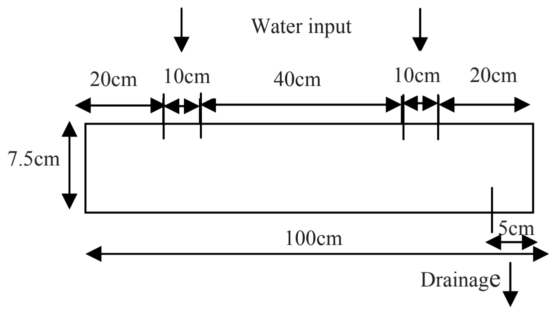


Fig. 1. Geometry of the growing substrate.

2.2. Theory

All Water movement in a porous media obeys the law of conservation of matter. Mathematically the continuity equation of incompressible water in rigid porous media in two dimensions is given as:

$$\frac{\partial \theta}{\partial t} = -\nabla.(\theta\,\mathrm{v}) - S_w = -\nabla.q - S_w \tag{1}$$

Where θ is the volumetric water content (L^3L^{-3}), t is the time (T), ∇ is the divergence operator (L^{-1}), v is the velocity of the water (LT^{-1}), $q = \theta.\mathrm{v}$ is the volumetric water flux density ($L^3L^{-2}T^{-1}$), S_w is the volumetric sink here considered as root uptake of water ($L^3L^{-3}T^{-1}$), x is an horizontal coordinate (L), and z is the vertical coordinate assumed positive downward (L) .

The dependent variables θ, v, q and S_w are functions of the independent variables x, z and t.

Darcy [10] obtained experimentally, for saturated porous media that q is proportional to the gradient in water potential. Buckingham [11] extended Darcy's law to partially saturated porous media, with the water potential expressed as a head equivalent, the law is given as:

$$q = -K(\theta)\nabla H \tag{2}$$

Where K(h) is an hydraulic conductivity of the porous media (LT^{-1}), ∇ is the gradient operator (L^{-1}), H is the hydraulic head (L) being the sum of the pressure head and the gravitational head z_g (L):

$$H = h(\theta) - z_g \tag{3}$$

The minus sign results from z taken positive downwards and the direction of flow is opposite to the gradient in total head.

The dependent variables $K, H,$ and h are functions of the independent variables x, z and t.

Richards [7] obtained by combination of continuity equation and Darcy's law the general governing flow equation for incompressible water transport in porous media. The h based form of Richards equation in two dimensions is given as:

$$C(h)\frac{\partial h}{\partial t} = \frac{\partial}{\partial x}\left(K(h)\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial z}\left(K(h)\frac{\partial h}{\partial z}\right) - \frac{\partial K(h)}{\partial z} - S_w \tag{4}$$

Where C(h) is the capillary capacity defined as $C = d\theta/dh$ (L^{-1})

2.3. Hydraulic properties

The hydraulic properties have been described by the models of Mualem [12] and Van Genuchten [13]. The hydraulic conductivity characteristic given by Mualem [12]:

$$K(h)=K_S\begin{cases}\dfrac{\left[\left(1+|\alpha h|^n\right)^m-|\alpha h|^{n-1}\right]^2}{\left(1+|\alpha h|^n\right)^{(\lambda+2)m}} & h\le 0\\ 1 & h>0\end{cases}\tag{5}$$

Where K_s is an hydraulic conductivity at saturation (LT^{-1}), λ, n, mare curve shape parameters (dimensionless), (with m=1-1/n), α is the curve shape parameter (L^{-1}).

The water retention characteristic given by the Van Genuchten function [13]:

$$S_e(h)=\frac{\theta(h)-\theta_r}{\theta_s-\theta_r}=\begin{cases}\dfrac{1}{\left(1+|\alpha h|^n\right)^m} & h\le 0\\ 1 & h>0\end{cases}\tag{6}$$

Where S_e is an effective saturation $0 \le S_e \le 1$, θ_r is the residual volumetric water content (L^3L^{-3}), θ_s is the saturated volumetric water content (L^3L^{-3}).

The capillary capacity $C = d\theta/dh$, deduced from the characteristic curve of water, is given by Hainen [14]:

$$c(h)=\begin{cases}(\theta_S-\theta_r)nm\alpha^n|h|^{n-1}\left(1+|\alpha h|^n\right)^{-1-m} & h\le 0\\ 0 & h>0\end{cases}\tag{7}$$

Experimentally [5] obtained for the rockwool the following physical properties:

K_s= 0.002 ms^{-1}, θs= 0.975 m^3m^{-3}, θ_r = 0.026 m^3m^{-3}, α = 15.917 m^{-1}, n = 3.061, m = 0.673

2.4. Boundary conditions and initial condition

The initial condition is that of given initial pressure head distribution $(h=-0.5cm)$.

The boundary conditions are as follows:

- Top: the condition is that of no flow $(q_z=0)$, except at the positions where the irrigation rate was set, to 2l/h (dripper's value) on an area equivalent to the base of the planting cube (100cm^2).
- Bottom: impermeable $(q_z=0)$ except at the drain location where the condition is:
  Substrate saturated: $q_z \ge 0 \quad h=0$
  Substrate unsaturated: $h<0 \quad q_z=0$
- Left and right: because of symmetry there is no flow across these boundaries $(q_x=0)$ or $\dfrac{\partial h}{\partial x}=0$.

2.5. Numerical implementation

Richards equation in two dimensions Equation (4) with equations (5), (6), and (7) is solved numerically using the control volume finite element method [15], after rearrangement of terms the numerically approximated form is:

$$Ah_{I,J}^{k+1} = A_W h_{I-1,J}^{k+1} + A_E h_{I+1,J}^{k+1} + A_N h_{I,J-1}^{k+1} + A_S h_{I,J+1}^{k+1} + b \tag{8}$$

$$A_W = \frac{K_{x,i-1,j}^{k}\Delta z_J}{\Delta x_{i-1}} \qquad A_E = \frac{K_{x,i,j}^{k}\Delta z_J}{\Delta x_i}$$
$$A_N = \frac{K_{x,i,j-1}^{k}\Delta x_I}{\Delta x_{j-1}} \qquad A_S = \frac{K_{x,i,j}^{k}\Delta x_I}{\Delta z_j} \tag{9}$$

$$b = A_C h_{I,J}^{k+1} - \left[K_{z,i,j}^{k} - K_{z,i,j-1}^{k}\right]\Delta x_I - S_{w,I,J}\Delta x_I \Delta z_J \tag{10}$$

Where:

$$A_C = C_{I,J}^{k}\frac{\Delta x_I \Delta z_J}{\Delta t} \tag{11}$$

$$A = A_E + A_W + A_N + A_S + A_C \tag{12}$$

The numerical solution gives for any t the θ(x,z) and h (x,z) distributions.

3. Results and discussion

Numerical calculations obtained by this model were compared with the results developed by [3] and show similar distributions of humidity for the substrate see Fig 2 and we observe that:

- The saturated zones are always located just below the drippers of the substrate;
- The saturated zones are always located at the base of the substrate;
- The area just above the drainage slot is always dry.

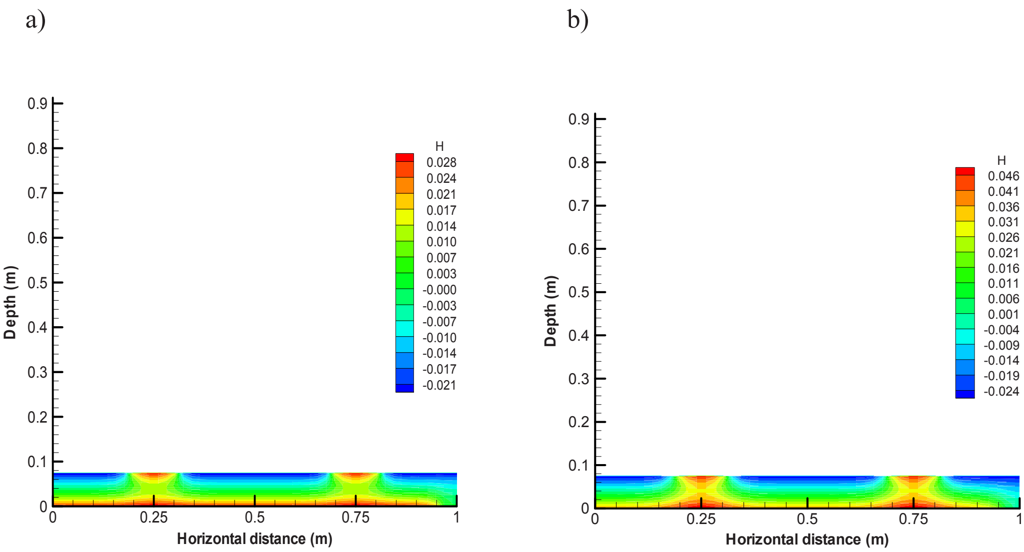


Fig. 2. Scenarios of water distribution a) beginning of the irrigation b) end of the irrigation

- In the higher layers the pressure head is maximum below the drippers and minimum otherwise see Fig 3.

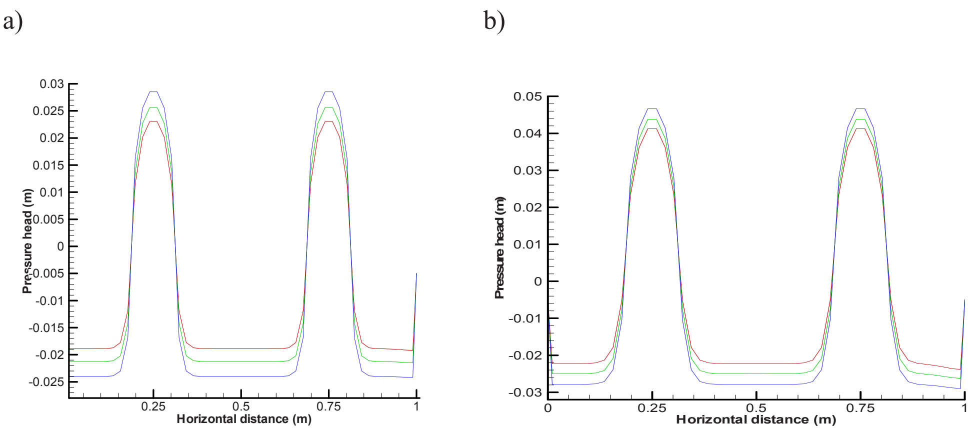
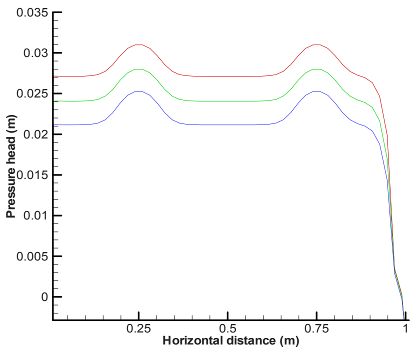


Fig. 3. Pressure head following the length of the substrate in the higher layers
a) beginning of the irrigation b) end of the irrigation.

- In the lower layers the pressure head is maximum below the drippers and reach mean values otherwise see Fig 4.

a) b)



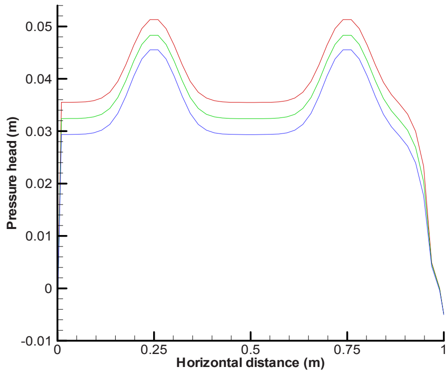


Fig. 4. Pressure head following the length of the substrate in the lower layers
a) beginning of the irrigation b) end of the irrigation

4. Conclusion

The simulation model can largely reduce the number of experiments and helps in finding alternative strategies. After the determination of hydraulic and physical proprieties of the rockwool slab using as growing substrate: Floriculture [5], these values are introduced for simulating the water movement [3] by the use of Computer Dynamics software [4].

In this study a two dimensional numerical model is developed to investigate the water movement in a rockwool slab by solving implicitly the Richards equation [7] for unsaturated porous medium, with the control volume finite element method. Scenarios of water distribution are presented compared with numerical results obtained by CFD which are validated by experimental results, in the higher layers the pressure head is maximum below the drippers only, but in the lower layers the pressure head is maximum in all the base of the substrate.

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